647510 **BSc3MatC301x** **Seat No :________**

B.Sc. Semester - 3 (*CBCS*) Examination
Oct/Nov - 2022 [NEW COURSE]
Mathematics-M301(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1(A) Answer any two out of three. **[10]**

1. State and prove necessary and sufficient condition for a non-empty subset W of a vector space V to be a subspace of V.
2. V is vector space. For $\overline{v_1}, \overline{v_2}, \overline{v_3}, \ldots \overline{v_n} \in$ V and SP{$\overline{v_1}, \overline{v_2}, \overline{v_3}, \ldots \overline{v_n}$}=V if $\overline{w_1}, \overline{w_2}, \overline{w_3}, \ldots \overline{w_n}$ are linearly independent vectors of V then prove that $m \leq n$.
3. Set A={(1,-2,5),(2,1,-1),(3,-1,x)}is subset of vector space R^3. If set A is linearly independent then find value of x.

Q-1(B) Answer any one out of two. **[04]**

1. Check whether vector (0,-1,1) ∈ SPA. where set A={(2,1,0),(-1,0,1),(0,1,2)}.
2. Check whether set A={sin(x), cos(x),cos(x-$\frac{\pi}{4}$)} of real vector space C[0,2π] is linearly dependent or linearly independent.

Q-2(A) Answer any two out of three. **[10]**

1. Prove that there exists a basis for each finite dimensional vector space V.
2. State and prove extension theorem for basis of vector space.
3. W_1={x-y+z,x+y-z,y+z,2x+4y+4z / x,y,z $\in$ R}
   W_2={2y-4z+w,y+z,y-z,w / y,z,w $\in$ R} be two subspaces of R^4.
   Find dim(W_1+W_2), dimW_1, dimW_2, dim($W_1 \cap W_2$).

Q-2(B) Answer any one out of two. **[04]**

1. V be a finite dimensional vector space then prove that any two basis of V have the same number of vectors.
2. Prove that set {1-x, 1+x,1-x^2} is base of P_2(R). Find coordinates of 3+7x+2x^2 with respect to this base.

Q-3(A) Answer any two out of three. **[10]**

1. if $\overline{f}$ and $\overline{g}$ are vector functions and ϕ is a scalar function defined on the domain D whose first order partial derivatives exists then prove that
   (i) curl($\overline{f}+\overline{g}$)= curl $\overline{f}$ + curl $\overline{g}$.
   (ii) curl($\phi\ \overline{f}$)=ϕ (curl $\overline{f}$)+(gradϕ) x $\overline{f}$.
2. if $\overline{f}$ and $\overline{g}$ are irrotational functions on D then show that $\overline{f}$ x $\overline{g}$ is a Solenoidal function.
3. Prove that $\frac{1}{r}$ satisfies the Laplacian equation. Also prove that if r = |r| Then Curl(gradr^n)= $\overline{0}$. Where $\overline{r}$ = (x,y,z).

Q-3(B) Answer any one out of two. **[04]**

1. if $\overline{f}$ = (x^3,y^3,z^3) then prove that curl $\overline{f}$ = $\overline{0}$ and grad(div $\overline{f}$)= 6$\overline{r}$.
2. in usual notation prove that div($r^n\overline{r}$) = (n+3) r^n

Q-4(A) Answer any two out of three. **[10]**

1. Find the area of circle whose radius is a by double integral.
2. Evaluate: $\iiint_R \frac{dxdy}{\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}}$ where R is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
3. Find the volume of a sphere having radius a.

Q-4(B) Answer any one out of two. **[04]**

1. Change the order of integration in the integral $\int_0^1 \int_{\frac{x^2}{a}}^{2a-x} f(x,y)dydx$ and hence Evaluate it.
   Where $f(x,y) = xy$
2. Change in spherical Coordinate $\iiint_R (x^2 y)dxdydz$. Where R is $x^2+y^2+z^2 \le a^2$.

Q-5(A) Answer any two out of three. **[10]**

1. State and prove Green's theorem for a plane.
2. Verify divergence theorem for $\overline{F} = 4x\bar{i} - 2y^2\bar{j} + z^2\bar{k}$ Where R is $x^2+y^2 = 4$, z=0 and z=3.
3. Define Beta function. Prove that $\beta(p,q) = \int_0^\infty \frac{y^{p-1}}{(1+y)^{p+q}}dy$. Where p,q > 0.

Q.5(B) Answer any one out of two. **[04]**

1. Evaluate: $\int_0^{\frac{\pi}{2}} (\sin^5\theta)(\cos^6\theta)d\theta$.
2. Verify stokes theorem for $\overline{F} = x^2\bar{i} + xy\bar{j}$. Where S is a square whose sides are x=0, y=0, x=a, y=a. where z=0.

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